PDEs and Relativistic Quantum Mechanics

Nice, May 11th-13th, 2022

Program

Wednesday, May 11th

- 9 : 00 – 9 : 55 Nicolas RAYMOND
  Semiclassical analysis of the Neumann Laplacian with constant magnetic field in three dimensions
- 10 : 00 – 10 : 55 Albert MAS
  Spectral analysis of a confinement model in relativistic quantum mechanics
- 11 : 00 – 11 : 30 Coffee break
- 11 : 30 – 12 : 25 Thomas OURMIÈRES-BONAFOS
  Spectral properties of relativistic quantum waveguides
- Lunch
- 15 : 30 – 16 : 00 Coffee break
- 16 : 00 – 16 : 25 Enguerrand LAVIGNE-BON
  Semiclassical spectrum of the Dirichlet-Pauli operator on an annulus
- 17 : 00 – 17 : 55 Fabio PIZZICHILLO
  Boundary value problems for 2-D Dirac operator on corner domains and the Coulomb interaction
- 20 : 00 Conference dinner

Thursday, May 12th

- 9 : 00 – 9 : 55 William BORRELLI
  Some properties of cubic Dirac equations in honeycomb structures
- 10 : 00 – 10 : 55 Annalaura STINGO
  Global stability of Kaluza-Klein Theories
- 11 : 00 – 11 : 30 Coffee break
- 11 : 30 – 12 : 25 Martin OELKER
  On domain, self-adjointness, and spectrum of two-body Dirac operators with interaction
- Lunch
- 15 : 30 – 16 : 00 Coffee break

- 16 : 00 – 16 : 25 Long MENG
  Dirac-Fock model: The structure and rigorous justification
- 17 : 00 – 17 : 55 Sebastian HERR
  Nonlinear Dirac Equations

Friday, May 13th

- 9 : 00 – 9 : 55 Yvonne ALAMA BRONSARD
  On low-regularity numerical approximations via decorated tree series
- 10 : 00 – 10 : 55 Ngoc Nhi NGUYEN
  Fermionic semiclassical L^p estimates
- 11 : 00 – 11 : 30 Coffee break
- 11 : 30 – 12 : 25 David CHIRON
  Smooth branches of travelling waves for NLS in 2d
- Lunch

Abstracts

Yvonne ALAMA BRONSARD, Sorbonne Université

On low-regularity numerical approximations via decorated tree series

In this talk I will discuss new time discretization techniques based on decorated tree series for solving a general class of nonlinear dispersive equations up to arbitrary order, and in low-regularity spaces. I will illustrate this decorated tree formalism in the context of the Gross-Pitaevskii equation. In the second part of my talk, I will present an error analysis result for solving the Gross-Pitaevskii equation at low-regularity, and show first and second order convergence in any fractional positive Sobolev space H^r, $r > 0$. These new schemes, together with their optimal local error estimates, allow for convergence under lower regularity assumptions than required by classical methods. The first part of this talk is joint work with Katharina Schratz (LJLL, Sorbonne Université), and Yvain Bruned (University of Edinburgh).

William BORRELLI, Università Cattolica del Sacro Cuore

Some properties of cubic Dirac equations in honeycomb structures

In this talk I will present recent results on some stationary 2d Dirac equations with cubic nonlinearities. They appear as effective models in two-dimensional honeycomb structures, under suitable assumptions.

More precisely, *massless* Dirac equations formally describe to leading order solutions to cubic NLS equations with honeycomb potential. This is related to the conical degeneracy of the spectrum of the associated honeycomb Schrödinger operator. It is known that suitable perturbations can open a gap at conical points, resulting in a *mass term* in the Dirac equation.

I will present some existence and multiplicity results for the effective Dirac model, both in the massive and massless case, and provide some qualitative properties. The main difficulty here is due to the fact that the equations are Sobolev-critical, which makes the existence and regularity arguments not straightforward. Our proofs rely on variational and shooting methods, while regularity follows by refined bootstrap arguments.

David CHIRON, Université Côte d'Azur

Smooth branches of travelling waves for NLS in 2d

We consider the travelling waves solutions for the nonlinear Schrödinger equation (NLS) in 2d with condition of modulus 1 at infinity. Several existence results for these waves have been obtained by variational method, parametrized by the momentum. We shall present some existence results of smooth branches parametrized by the speed corresponding to the asymptotic behaviours where the wave has two vortices -joint works with E. Pacherie- or is a rarefaction pulse described by the lump solitary wave of KP-I.

Sebastian HERR, Universität Bielefeld

Nonlinear Dirac Equations

First, I will review the dispersive estimates which have been developed to prove global well-posedness and scattering for cubic Dirac equations in critical spaces. Then, I will describe recent progress on bilinear Fourier restriction estimates. Finally, I will outline how these can be used to give a unified proof of well-posedness and scattering for both massive and massless cubic Dirac equations.

Enguerrand LAVIGNE-BON, Aix-Marseille Université

Semiclassical spectrum of the Dirichlet-Pauli operator on an annulus

We are interested in the stationary states of the Pauli equation. This equation is the formulation of the Schrödinger equation for non-relativistic spin-1/2 particles subjected to a magnetic field. Recent work has focused on the semi-classical limit of the low-lying spectrum when the magnetic field is assumed to be strictly positive. We continue this study in the annulus case, when the magnetic field is assumed to be radial. By combining the new proof strategy introduced in by Barbaroux et al. (case of simply connected domains) with a reinterpretation of the ideas developed by Helffer and Persson we will see how the topological type of the domain plays a role in the semi-classical study of the spectrum of this operator.

Albert MAS, Universitat Politècnica de Catalunya

Spectral analysis of a confinement model in relativistic quantum mechanics

In this talk we will focus on the Dirac operator on domains of $\mathbb{R}^3$ with confining boundary conditions of scalar and electrostatic type. This operator is a generalization of the MIT-bag operator, which is used as a simplified model for the confinement of quarks in hadrons that has interested many scientists in the last decades. It is conjectured that, under a volume constraint, the ball is the domain which has the smallest first positive eigenvalue of the MIT-bag operator. I will describe our results – in collaboration with N. Arrizabalaga (U. País Vasco), T. Sanz-Perela (U. Autónoma de Madrid), and L. Vega (U. País Vasco and BCAM) – on the spectral analysis of the generalized operator. I will discuss on the parameterization of the eigenvalues, their symmetry and monotonicity properties, the optimality of the ball for large values of the parameter, and the connection to boundary Hardy spaces.

Long MENG, CERMICS, École des ponts ParisTech

Dirac-Fock model: The structure and rigorous justification

In this talk, we study the relationship between Dirac–Fock energy and electron-positron Hartree-Fock energy. We prove the longstanding conjecture due to Mittleman: the Dirac–Fock model is an approximation of QED when the fine structure constant α is small and the

velocity of light c is large. We also prove some properties about Dirac–Fock model. The proof is based on deep insight into the properties of Dirac–Fock model.

Ngoc Nhi NGUYEN, Université Paris-Saclay

Fermionic semiclassical L^p estimates

Spectral properties of Schrödinger operators are studied a lot in mathematical physics. They can give the description of trapped fermionic particles. Researches on the spatial concentration of semiclassical Schrödinger operators' eigenfunctions are still carried out. There are very precise results in special cases like the harmonic oscillator. However, it is not always possible to obtain explicitly pointwise information for more general potentials. We can measure the concentration by estimating these functions with L^p bounds.

Martin OELKER

On domain, self-adjointness, and spectrum of two-body Dirac operators with interaction

Interest in the mathematical treatment of two-body Dirac operators H_{2BD}, that describe two interacting electrons, arose from relativistic quantum chemistry. There, the existence of square-integrable eigenfunctions of such operators is assumed without further justification. In this talk, we discuss a self-adjoint extension that is uniquely distinguished by means of finite potential energy. The difficulties we encounter are the unboundedness of H_{2BD} from below, and that the interaction potential is not relatively bounded by the free Hamiltonian. We also address the question of absence of eigenvalues for energies larger than twice the electron's mass. If time allows, we also investigate the domain of H_{2BD} and the possibility of infinite single particle kinetic energy states in this domain.

Thomas OURMIÈRES-BONAFOS, Aix-Marseille Université

Spectral properties of relativistic quantum waveguides

In this talk I will discuss recent results on the spectrum of the Dirac operator posed in a tubular neighborhood of an unbounded planar curve subject to infinite mass boundary condition. After reviewing well known results on the non-relativistic counterpart of this hamiltonian we will particularly investigate the thin waveguide regime and exhibit the existence of a one dimensional effective operator.

This is joint work with: W. Borrelli, P. Briet, D. Krejcirik.

Fabio PIZZICHILLO, Universidad de Cantabria

Boundary value problems for 2-D Dirac operator on corner domains and the Coulomb interaction

This talk aims to present results on the self-adjoint extensions of Dirac operators on plane domains with corners in dimension two. We consider the case of infinite-mass boundary conditions and we obtain explicitly the self-adjoint extensions of the operator. It turns out that the presence of corners typically spoils the elliptic regularity known to hold for smooth boundaries.

Then we discuss the self-adjointness and some spectral properties of these operators in presence of a Coulomb-type potential with the singularity placed on the vertex.

This is a collaboration work with Hanne Van Den Bosch, Biagio Cassano and Matteo Gallone.

Nicolas RAYMOND, Université d'Angers

Semiclassical analysis of the Neumann Laplacian with constant magnetic field in three dimensions

This talk deals with the spectral analysis of the semiclassical Neumann magnetic Laplacian on a smooth bounded domain in dimension three. When the magnetic field is constant and in the semiclassical limit, we establish a four-term asymptotic expansion of the low-lying eigenvalues, involving a geometric quantity along the apparent contour of the domain in the direction of the fi eld. In particular, we prove that they are simple under generic assumptions and we are led to revisit the two-term expansion of the lowest eigenvalue obtained by Helffer and Morame in 2004. Joint work with Frédéric Hérau.

Annalaura STINGO, École Polytechnique

Global stability of Kaluza-Klein Theories

The Kaluza-Klein theories represent the classical mathematical approach to the unification of general relativity with electromagnetism and more generally with gauge fields. In these theories, general relativity is considered in 1+3+d dimensions and in the simplest case d=1 dimensional gravity is compactified on a circle to obtain at low energies a (3+1)-dimensional Einstein-Maxwell-Scalar systems. Kaluza-Klein space-times have been proven by Witten to be unstable at the semi-classical level, but heuristic arguments are given in support of their classical stability.

In this talk I will discuss the problem of the classical global stability of Kaluza-Klein theories when d=1 and present a toy model we studied in collaboration with C. Huneau.