Workshop "The analysis of Dirac equations"

Orsay, June 6th-8th, 2018

The talks will take place in room 2L8 at the Institut de Mathématique d'Orsay (Building 307).

Program

Wednesday, June 6th

- 14 : 00 – 14 : 55 Mathieu LEWIN **Self-adjointness and min-max formulae for Dirac operators with Coulomb type potentials**
- 15 : 00 – 15 : 55 Mathieu LEWIN **Self-adjointness and min-max formulae for Dirac operators with Coulomb type potentials**
- 16 : 00 – 16 : 30 Coffee break (Room 2P8)
- 16 : 30 – 17 : 25 Jean-Claude CUENIN **Some bound state problems for Dirac operators**

Thursday, June 7th

- 9 : 30 – 10 : 25 Philippe GRAVEJAT **Effective models for electromagnetic fields in the Dirac vacuum**
- 10 : 30 – 11 : 00 Coffee break (Room 2P8)
- 11 : 00 – 11 : 55 Naiara ARRIZABALAGA **Spectral properties for the MIT bag model**
- 12 : 00 – 12 : 55 Thomas OURMIÈRES-BONAFOS **Dirac operators and delta interactions**
- Lunch
- 15 : 30 – 16 : 25 Timothy CANDY **Global existence for the Dirac-Klein-Gordon equation**
- 16 : 30 – 17 : 00 Coffee break (Room 2P8)
- 17 : 00 – 17 : 55 Jérémy SOK **Dirac operators with magnetic links**
- 18 : 30 Cocktail and conference dinner (Room 2P8)

Friday, June 8th

- 9 : 30 – 10 : 25 Nabile BOUSSAID **On stability of solitary waves of the nonlinear Dirac equation in the non-relativistic limit**
- 10 : 30 – 11 : 00 Coffee break (Room 2P8)
- 11 : 00 – 11 : 55 Anne-Sophie DE SUZZONI **Dispersion and the Dirac operator**
- 12 : 00 – 12 : 55 Federico CACCIAFESTA **Strichartz estimates for the Dirac equation on spherically symmetric manifolds and related problems**

Abstracts

Mathieu LEWIN, CNRS & Université Paris-Dauphine

Self-adjointness and min-max formulae for Dirac operators with Coulomb type potentials

In this talk I will review some well known spectral properties of Dirac operators with Coulomb type potentials. I will in particular discuss self-adjointness and what is known about its domain. In the second part I will present some recent results on min-max formulas for its eigenvalues, obtained in collaboration with Maria Esteban and Eric Séré (Paris-Dauphine).

Jean-Claude CUENIN, Ludwig-Maximilians-Universität München

Some bound state problems for Dirac operators

I will discuss several results related to bound state problems for self-adjoint and non-self-adjoint Dirac operators on the whole Euclidean space or on a two-dimensional waveguide. The latter serves as a model for charge carriers in a graphene strip. I will discuss existence and asymptotics of weakly coupled bound states, dipole potentials, Lieb-Thirring type bounds and examples of embedded eigenvalues.

Philippe GRAVEJAT, Université de Cergy-Pontoise

Effective models for electromagnetic fields in the Dirac vacuum

We present recent results concerning two effective models for the polarization of the quantum vacuum by electromagnetic fields: the Pauli-Villars model, which provides a rigorous description of polarization when the fields are supposed to be classical, and the Euler-Heisenberg model, which we derive in a purely magnetic, weakly varying regime.

Naiara ARRIZABALAGA, University of the Basque Country

Spectral properties for the MIT bag model

In this talk we present different spectral results for the MIT bag model. This model is the Dirac operator, $-i\alpha \cdot \nabla + m\beta$, defined on a smooth and bounded domain of $\mathbb{R}^3$, Ω, with certain boundary conditions. Specifically, $-i\beta(\alpha \cdot n)\psi = \psi$ must hold at the boundary of Ω, where n is the outward normal vector and $\psi \in H^1(\Omega, \mathbb{C}^4)$. It was developed as a simplified model for a relativistic particle confined in a box, also called the bag, Ω. And, it has been very fruitful since it allowed to predict the existence of many particles. We will see the MIT bag model as a shell interaction for Dirac operators. In particular, we will show how the MIT bag model is related with the Dirac operator with electrostatic and Lorentz scalar shell potentials. Then, we will study the point spectrum and confinement for those last operators. And finally, we will describe the limiting behavior of the eigenvalues of the MIT bag model as the mass m tends to $\pm\infty$.

In this talk I will bring together two works, one of them is a joint work with A. Mas and L. Vega, and the other one, with L. Le Treust and N. Raymond.

Thomas OURMIÈRES-BONAFOS, Université Paris Sud

Dirac operators and delta interactions.

In this talk, we will discuss different aspects of the Dirac operator in dimension three, coupled with a singular potential supported on a surface. After motivating the study of such objects, we will briefly be interested in the problem of self-adjointness for singular electrostatic or Lorentz-scalar potentials. For this last class of potentials, we will study the structure of the spectrum of such an operator and in particular, we will show that for an "attractive" potential, when the mass of the particle goes to infinity, the behavior of the eigenvalues is given by an effective operator on the surface. We will see that this effective operator is actually a Schrödinger operator with both a Yang-Mills potential and an electric potential, each one being of geometric nature.

These are joint works with Markus Holzmann, Konstantin Pankrashkin and Luis Vega.

[1.] A strategy for self-adjointness of Dirac operators: applications to the MIT bag model and delta-shell interactions, with Luis Vega, 30p., to appear in Publicacions Matemàtiques, arXiv:1612.07058.
[2.] Dirac operators with Lorentz scalar shell interactions, with Markus Holzmann and Konstantin Pankrashkin, 41 p., to appear in Reviews in Mathematical Physics, arXiv:1711.00746.

Timothy CANDY, Universität Bielefeld

Global existence for the Dirac-Klein-Gordon equation

The question of large time behaviour for semilinear Dirac models is particularly delicate due to the lack of good conserved quantities. Thus the current understanding of the asymptotic behaviour is largely restricted to small data regimes. In this talk we will review the small data theory for the Dirac-Klein-Gordon equation, and mention some recent (and not so recent) progress towards obtaining a class of large data solutions.

Jérémy SOK, Universität Basel

Dirac operators with magnetic links

The existence of zero modes for Dirac operators with magnetic fields is the cause of break down of stability of matter for charged systems.

However the known examples are geometrically complex, and a complete clas- sification of zero modes is unknown. In particular, one does not know the characteristics of the magnetic fields which produce the zero modes.

To better understand them, we studied the zero modes in the particular case of magnetic fields with finitely many field lines forming a link. These singular fields can be seen as generalizations of the Aharonov-Bohm solenoids, and they exhibit the same $2\pi-$periodicity of the fluxes carried by their field lines.

Tuning one flux from 0 to 2π gives rise to a loop of Dirac operators for which we can study the spectral flow, a non-trivial spectral flow indicating the occurence of zero modes. It turns out that this number depends on the geometry of the magnetic fields: the interlinking of the field lines but also their shape.

Joint work with Fabian Portmann and Jan Philip Solovej

Nabile BOUSSAID, Université de Franche-Comté

On stability of solitary waves of the nonlinear Dirac equation in the non-relativistic limit

This is a joint work with Andrew Comech. We study the point spectrum of the linearization at a solitary wave solution to the nonlinear Dirac equation with the soler nonlinear term. We focus on the spectral stability, that is, the absence of eigenvalues with positive real part, in the non-relativistic limit. We prove the spectral stability of small amplitude solitary waves for the "charge-subcritical cases" and for the "charge-critical case".

An important part of the stability analysis is the proof of the absence of bifurcations of nonzero-real-part eigenvalues from the embedded threshold points. Our approach is based on constructing a new family of exact bi-frequency solitary wave solutions in the Soler model, on using this family to determine the multiplicity eigenvalues of the linearized operator, and on the analysis of the behaviour of "nonlinear eigenvalues" (characteristic roots of holomorphic operator-valued functions).

Anne-Sophie DE SUZZONI, Université Paris 13

Dispersion and the Dirac operator

In this talk, I will present some aspects of dispersion for the Dirac operator. I will start by partially reviewing what is known for the Dirac operator in a Minkowkski space-time. Then, I will introduce the Dirac operator in a curved space-time, and present a result of dispersion for specific cases such as assymptotically flat or warped product geometries. This is a joint work with F. Cacciafesta (Padova).

Federico CACCIAFESTA, Università degli Studi di Padova

Strichartz estimates for the Dirac equation on spherically symmetric manifolds and related problems

In this talk we show how to exploit the "radial" structure of the Dirac operator to obtain dispersive estimates (local smoothing, Strichartz..) for the Dirac equation on spaces with spherical symmetry. As we will see, this problem is naturally related to the study of the Dirac dynamics perturbed with scaling critical potentials. Applications to the well-posedness for some nonlinear models will be also discussed. This is a joint (ongoing) work with A. S. de Suzzoni.